Analytical calculation of membranes and foils for building skins

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Abstract

A computer model is a discrete mathematical representation of the reality. It should enable the most accurate and complete formulation of a structure. If high-quality results must be achieved, software products must meet these requirements. Various methods and tools for generating accurate and complete models are shown. Fast and efficient form finding procedures are explained more detailed.

Static analysis is a challenge in case of a holistic formulation. The membrane/foil material behavior must be applied correctly, usually it is described by 4 values (E-modulus in warp and weft-direction, the crimp-stiffness and the shear modulus). Furthermore, the structure must be calculated in a hybrid system (membrane together with steel). In case of pneumatic systems, the statics can be done under 3 possible constraints: volume, internal pressure or gas-law (product of volume and inner pressure remain constant) are shown.

Cutting pattern theories are mentioned briefly, here we put the focus on flattening theories as it is the main part of the cutting pattern generation of prestressed double curved surfaces.

After the theoretical part, examples for form finding and cutting pattern generation of large pneumatic multi-layer ETFE cushion projects are described. The procedure for a parameterized automatic calculation is explained in more detail using the examples.

Keywords: Form finding, Statics, Cutting Patterns, Membrane, Foil

1. Introduction

Today, computer models play an important role in the calculation of textile membrane and foil structures. To be able to derive high-quality results from a model, the software used must enable the most accurate and complete description of a structure.

A great advantage of computer models is the possibility to perform a holistic calculation. Holistic in this context means that a complete model is analyzed under external loadings by taking into consideration any boundary condition. Computer models can also be used to automate processes. This can greatly accelerate the working cycle, especially for large structures.

Even though today's software tools offer considerable modelling possibilities, the user should always bear in mind that a computer model does not represent the exact reality and is always a partial and abstract view of reality.

2. Form finding

In a conventional design the architect fixes the real geometry on a drawing board. This procedure does not work for textile membrane and foil structures. This is not possible with respect to pre-stressed lightweight structures because internal forces or stresses and the surface geometry are not independent of each other. Therefore, when the usual design procedure is not possible, a form finding process is needed.

The form finding process can be described as the search for a geometry with ideal force flow and a favorable distribution of membrane stresses, considering aesthetic and constructional aspects.

For the form finding process you have in principle 2 possibilities. Either one builds physical models e.g. from nylon, soap skin, mesh, etc. or one uses analytical form finding methods.

Figure 1: Form finding with physical and computer model

The physical model has limitations regarding fast changes in the boundary conditions. The biggest problem, however, is the problem of scale: Inaccuracies in the model multiply with the scale of the model when transferred to reality, cutting patterns for textile membranes and cable lengths can generally not be derived from the physical models. Therefore, from the mid-1970s onwards, more and more analytical methods were used.

2.1 Analytical form finding

Analytical form finding theories are finite element methods. The surfaces are divided into several small finite elements as triangles for example. Most software programs use either the linear force density method or the nonlinear dynamic relaxation method. The force density method is a mathematical strategy for solving the equations of equilibrium for any type of cable network, without requiring any initial coordinates of the structure. The ratio of internal force to stressed length is defined as force density and assumed to be known. Thus, one obtains a linear system of equations which can be solved in one step without approximate values. Initially, the force density method was developed for cable nets, later extended to membranes and foils.

During the design process, the user must first specify the support points and their connections. Then, a first figure of equilibrium can be found by specifying the mesh direction and the magnitude of the prestress. By varying the initial values (mesh direction, prestress), the support definition (selective by point, linear as cable or arc, areal as internal pressure) and the arrangement of the boundary lines (flexible or rigid), the designer can vary the shape according to his ideas. In the case of pneumatic structures, the internal pressure or volume is used as an additional forming parameter.

2.2 Structural behavior of membranes and foils

Membranes and foils have a special structural behavior. The material is flexible and only tensile- (and shear-) stresses can be carried. We must use the membrane in a way that loads can be transferred only by tensile forces. This can be achieved by a double curved geometry and the introduction of prestress. Whatever the structural configuration, we need forms with double curvature to resist the applied loads. This can be of two types:

The surface of mechanically pre-stressed membranes is anticlastic doubly curved. For these types of surfaces an initial stiffness exists, all points can carry downward forces (e.g. snow) as well as upward forces (e.g. wind suction).

Surfaces of pneumatically pre-stressed membranes are synclastic doubly curved, pressure differences are used to introduce the pre-stress. The pneumatically stressed surface carries snow loads by reducing the membrane stress and increasing the internal pressure, while we find the opposite situation for wind suction.

Figure 2: Mechanically (left) and pneumatically (right) pre-stressed structures

3. Statics

A static calculation for membranes and foils is geometrically nonlinear. We need material properties for all elements and its nondeformed geometry. The nondeformed geometry of a cable element for instance is the unstressed length of this cable. Next, we need the external loads and in case of a pneumatic structure the internal pressure or volume information. After the form finding procedure a geometry is available and statics can be performed.

In contrast to foils, whose material properties are isotropic, orthotropic material properties must be applied to textile membranes. With isotropic material, the stiffness is the same in all directions or can be assumed to be the same. An orthotropic material has a special directional dependency in 2 perpendicular directions. In these directions there are different force-strain behaviors. The orthotropic directions are not independent of each other if the membrane crimp stiffness is different from 0.

In our software we use an extended material law. The stresses in warp and weft direction and the shear stresses are calculated as follows:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_u \\ \sigma_v \\ \tau \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \Delta\alpha \end{bmatrix} \qquad \text{(3-1)}$$

$$m_{12} = m_{21}$$

with:

σ_u	Stress in warp direction
σ_v	Stress in weft direction
τ	Shear stress
m_{12}	Membrane crimp stiffness
m_{11}	Membrane stiffness in 1000
m_{22}	Membrane stiffness in 2000
m_{33}	Membrane shear stiffness
ε_u	Strain in warp
ε_v	Strain in weft
$\Delta\alpha$	Shear deformation

In addition to the stresses in warp and weft direction, the shear stress can be calculated if the shear stiffness is introduced as material value.

In the case of pneumatic structures, additional conditions must be observed: While in mechanically tensioned structures the loads do not change their size and direction during the loading process, this is different in pneumatically tensioned structures. The loading (magnitude or direction) does depend on the displacements of the model, the load vector must be set up based on the current displacements.

Furthermore, it is important for a realistic behavior of the calculation model that the software used offers appropriate possibilities. We recommend 4 calculation methods for the static calculation of pneumatic structures:

1. Given internal pressure p (snow)
2. Given volume V (water)
3. Given product $p{\cdot}V$ (Boyle-Mariotte, for example wind)
4. Given product $\frac{p{\cdot}V}{T}$ (General gas equation, consideration of temperature)

The third mode (consideration of gas-laws) enables the realistic behavior of the internal pressure. This mode is important in case of e.g. fast wind gusts. Here the pump systems cannot update the inner pressure in the short time. We can see it as a closed system and by considering the temperature as constant we get the gas law of Boyle and Mariotte $p{\cdot}V{=}const$ in this case. Only if the gas law is fulfilled the membrane stresses get the correct size. Mode 4 also considers the temperature, the principle itself is the same as mode 3.

The modes can be used e.g. as follows:

1. An air hall under snow-loading (a specific internal pressure is set to resist the snow-loads)
2. A membrane filled with an incompressible fluid (water-bag) and
3. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($p{\cdot}V{=}const$) is valid.
4. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($\frac{p{\cdot}V}{T}{=}const$) is valid.

To get correct results software packages should consider these effects.

4. Holistic calculation

To obtain the most realistic values for the static behavior of a membrane structure a computer model should enable the most accurate and complete formulation of a structure. With the next two sections we would like to point out special aspects in the calculation of prestressed membrane and foil structures combined with bending stiff primary structures.

4.1 Comparison - separated systems and combined systems

In membrane engineering often the membrane system and the primary steel structure are calculated in separated systems by loading the steel elements with the reaction forces of the pneumatic membrane which was fixed at its boundaries in a first calculation step. This procedure is wrong with respect to form finding and statics. Separation is only allowed if the deflections are very small and this is never the case for membrane structures. In case of statics it is only a first - very imprecise (and expensive) estimation. Users should always calculate with computer models consisting of primary structure and membrane in one holistic model.

In the following real example, we show the differences using concrete numerical values. The example shows a chambered pneumatic structure with a bending stiff ring.

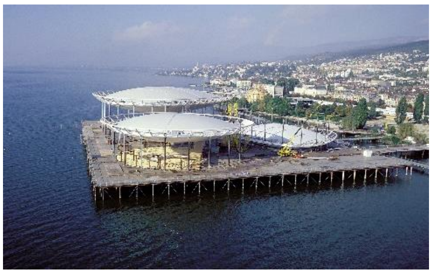


Figure 3: "Galets" for the Expo 2002, Neuchatel

Such structures are often divided into 2 subsystems. The first system is the pneumatic membrane, the second system describes the primary structure with the steel ring and the struts with cables.

Figure 4: Divided overall system. Left membrane, right steel ring with supports and cables.

The reaction forces from the membrane calculation are then applied as external loads on the primary structure in a second step. The design is then carried out using the results from the structural analysis. In the concrete example, the maximum deformation was 0.5 m and a maximum bending moment of 30000 kNm.

If the system is calculated holistically (membrane and primary structure in one system), we obtain 0.25 m for the maximum deformation. The maximum bending moment is 18000 kNm.

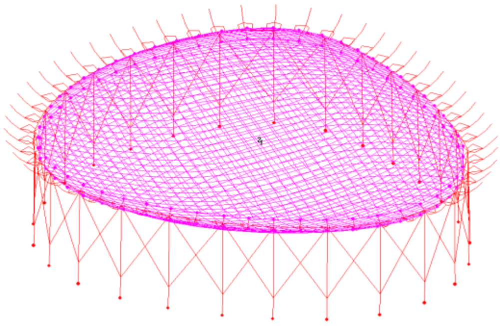


Figure 5: Membrane- and primary structure in a holistic system

The separation of nonlinear lightweight structures into subsystems is only a first, very inaccurate and expensive estimation. This fact applies to all membrane and foil structures, regardless of whether they are used as pure roof support structures or as building skins.

4.2 Form finding considering bending elements

In membrane and foil structures, prestress is of central importance; without prestress the supporting structures are not viable. The engineer specifies the desired prestress in the form-finding process and must ensure that this prestress exists also in the static load case.

If the flexibility of the bending elements is not included in the form finding process but is considered in the static load case, we do not end up with the desired prestress. In this case, all further static load cases are calculated with a wrong model.

Let us first illustrate this fact with a flat steel frame in which a membrane is clamped. The same behavior can be observed with pneumatic cushions.

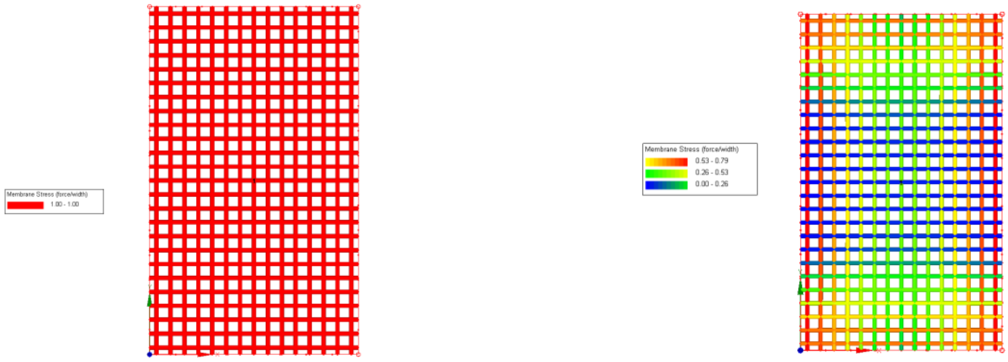


Figure 6: Form finding with fixed boundary and prestress 1:1 (left), Statics with free boundary and lower prestress (right)

It is therefore necessary to first calculate a mixed form finding. By this we mean the calculation of the force density-controlled membrane elements together with the elastically controlled bending stiff elements in one model. As a result, we obtain a deformed geometry with the desired prestress in the membrane or foil. Based on this result, new unstressed lengths for the membrane elements can be determined and transferred to the model.

In this way we obtain a static model in which the bending stiff elements and the membrane elements are elastically controlled and the desired prestress is maintained, although the bending elements deform.

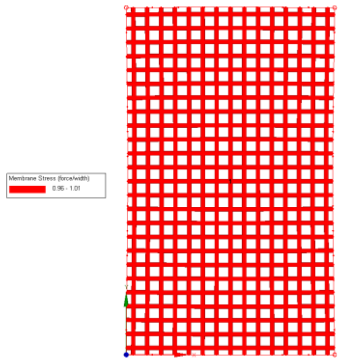


Figure 7:Statics with free boundary and prestress 1:1

The only difference between the frame (Figure 6 right and Figure 7) is that the cutting patterns of the foil are different.

In the following we show the influence of a wrong model on a frame grid. If you look at the stresses separately in x- and y-direction, you can clearly see that the stresses decrease towards the inside.

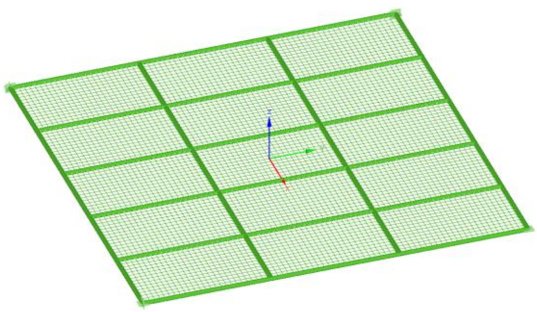


Figure 8: Frame grid 3x5, fields perspective view

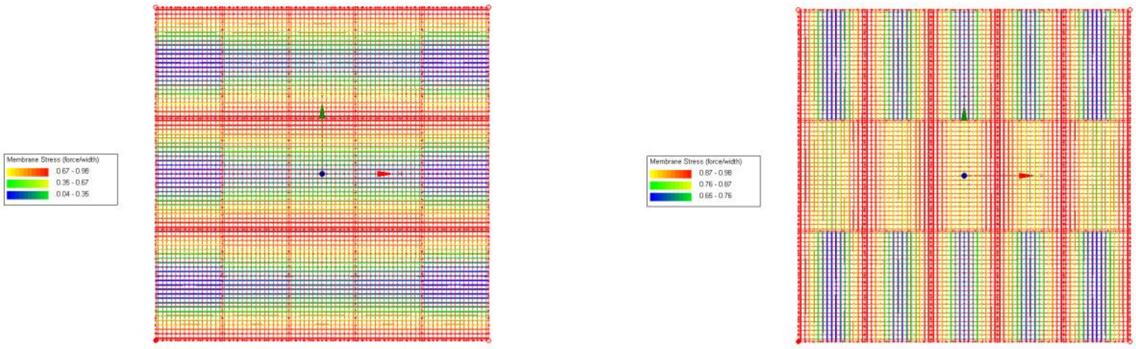


Figure 9: Wrong model and the obtained stresses in x-direction (left) and stress in y-direction (right)

The correct model gives the desired stresses of 1:1. We will refrain from using a graphic at this point.

5. Cutting pattern generation

The cutting pattern generation is an essential part of the technical process in textile architecture. It can be described as follows: Given the equilibrium shape of the curved membrane surface S, determine a set of n planar sub-surfaces {S_1, S_2, S_3,..., S_n} such that the distortion between S and S´ is minimized, where S´ is a surface of type S created by reassembling the sub-surfaces.

The task is therefore to bring a double-curved, pre-stressed surface onto a flat material of limited width in such a way that when the strips are welded and built up, exactly the shape previously modelled in the computer is created.

Cutting pattern generation is influenced by the following factors:

1.) Because double-curved surfaces cannot be mapped into the plane without distortion, efficient flattening strategies must be used.
2.) The planar strips must be as straight as possible to keep the cutting out waste as low as possible.
3.) The width of the 2D strips should be as wide as possible to minimize the amount of work. The maximum strip width depends on the roll width. Nevertheless, the distortions in the flattening process must be kept as small as possible (see 5.1).
4.) The geometrically developed surface must be corrected (compensated) to establish the prestressed surface.
5.) Corresponding seam lines must have the same length to avoid problems by joining the strips.

5.1 Surface flattening

To provide sufficient resistance against external loads membrane and foil structures have double-curvature. The double curvature gives the material sufficient stiffness to withstand the loads to which it is subjected. Therefore, the structural engineer aims to generate a form with a maximum double curvature to increase the load bearing capability efficiently. The larger the curvature in the surface, the more difficult it is to flatten the individual cutting strips to a maximum width with minimal distortion. The flattening strategy used therefore has a decisive influence on the quality of the cutting patterns.

In our software we use a method derived from map projection. The minimal distortion energy approach can be formulated as follows:

$$\Pi = \sum_{i=1}^{m} p_s((l_{o1} - l_1)^2 + (l_{o2} - l_2)^2 + (l_{o3} - l_3)^2) + p_\alpha(\alpha_0 - \alpha)^2 + p_A(A_0 - A)^2 \Rightarrow min. \qquad (5\text{-}1)$$

with:

$\Pi = \text{Distortion Energy}$ $p_\alpha = angle\ weight$ $p_s = distance\ weight$ $p_A = area\ weight$

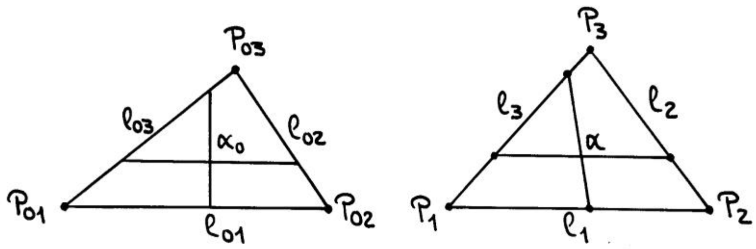


Figure 10: Non-deformed situation in 3d (left) and deformed situation in 2d (right)

5.2 Cutting patterns for pneumatic cushions

With pneumatic cushions, the connection to a supporting structure is made via a multi-part bending stiff frame profile. This means that there is no possibility for adjustment in case of inaccurate cutting patterns. Therefore, the cutting patterns must be very precise.

To avoid waste of material we must adjust the maximum patterning widths to the role widths. The maximum widths of cushion-patterns lie in the ridge line. Therefore, an automatic widths-optimization is possible using this line as guideline. For large projects, as much material as possible should be saved. The waste optimization is an economic factor for such projects. For pneumatic cushions, geodesic lines perpendicular to the ridgeline can be generated automatically and then the flattening can be performed as described above.

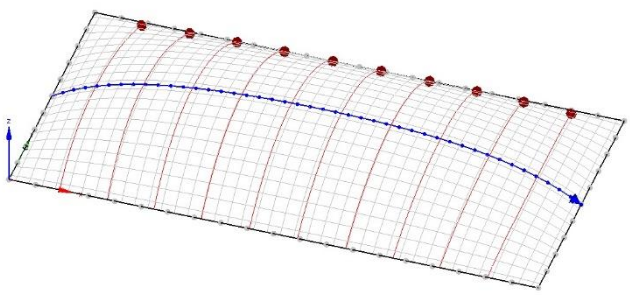


Figure 11: Ridge line as a guideline for geodesics

In the case of cushions with several layers, the position of the seam lines on the frame profiles plays an important role when generating cutting patterns. If all seam lines arrive at the same position on the frame, there will be difficulties during clamping because of the thickened material at this point. The software must provide automated methods that allow to set the seam line offsets.

6. Examples – Building skins with ETFE cushions

In the examples shown below, the techniques shown in the previous sections have been successfully applied.

The facade of the Allianz Stadium is one of the largest membrane shells in the world. It consists of 2,784 pneumatically pre-stressed cushions made of ETFE foil. Approximately half of the cushions are of different sizes and have a different shape. The cushions consist of one chamber and usually have a diamond-shaped edge.

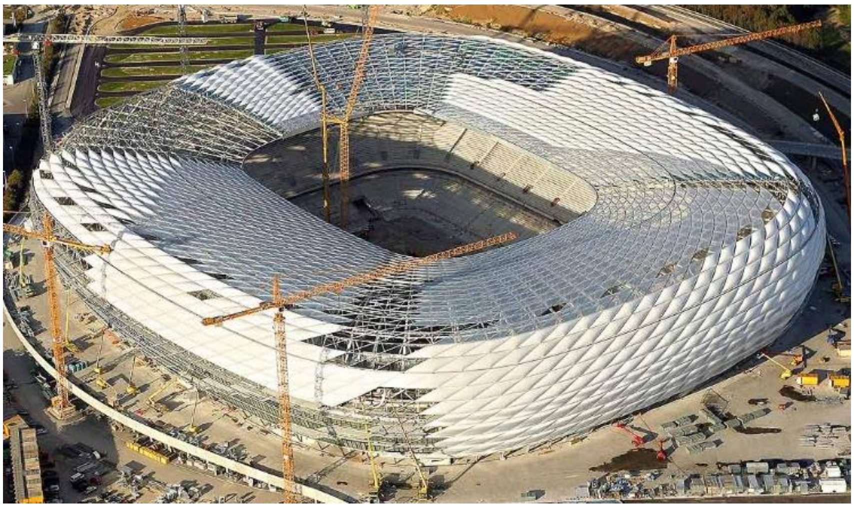
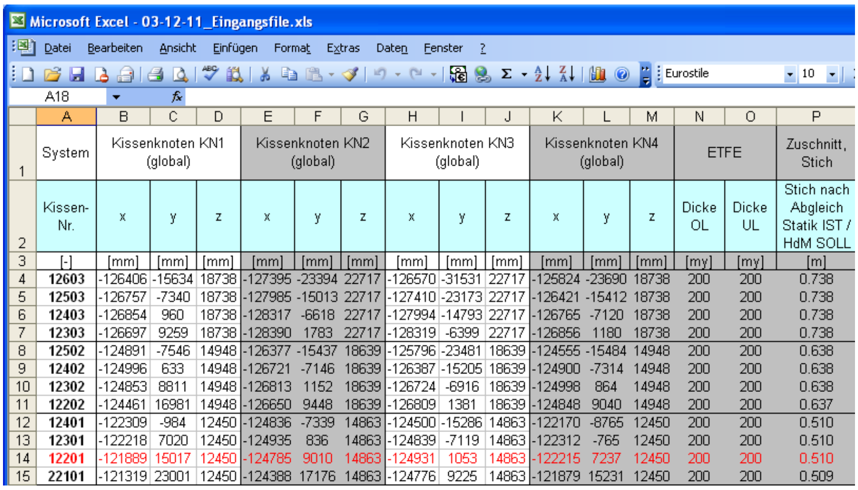


Figure 12: Allianz Arena Munich

Figure 13: Cushion mockups

Due to the large number of different cushions, it was not possible in this project to carry out the form finding and cutting pattern generation manually. To accelerate the calculation process automated methods were used. In this project the company covertex GmbH was responsible for the cushion facade, the company technet GmbH supplied the software for the automated calculation.

The parameters of the form finding for the individual cushions were specified in an Excel table, each row for one cushion. Together with the boundary geometry, the software then determined the equilibrium figures for the cushions automatically. A predefined sag was used as a break-off criterion.



Microsoft Excel - 03-12-11_Eingangsfile.xls

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	System	Kissenknoten KN1 (global)			Kissenknoten KN2 (global)			Kissenknoten KN3 (global)			Kissenknoten KN4 (global)			ETFE		Zuschnitt, Stich
2	Kissen-Nr.	x	y	z	x	y	z	x	y	z	x	y	z	Dicke OL	Dicke UL	Stich nach Abgleich Statik IST / HdM SOLL
3	[-]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[my]	[my]	[m]
4	**12603**	-126406	-15634	18738	-127395	-23394	22717	-126570	-31531	22717	-125824	-23690	18738	200	200	0.738
5	**12503**	-126757	-7340	18738	-127985	-15013	22717	-127410	-23173	22717	-126421	-15412	18738	200	200	0.738
6	**12403**	-126854	960	18738	-128317	-6618	22717	-127994	-14793	22717	-126765	-7120	18738	200	200	0.738
7	**12303**	-126697	9259	18738	-128390	1783	22717	-128319	-6399	22717	-126856	1180	18738	200	200	0.738
8	**12502**	-124891	-7546	14948	-126377	-15437	18639	-125796	-23481	18639	-124555	-15484	14948	200	200	0.638
9	**12402**	-124996	633	14948	-126721	-7146	18639	-126387	-15205	18639	-124900	-7314	14948	200	200	0.638
10	**12302**	-124853	8811	14948	-126813	1152	18639	-126724	-6916	18639	-124998	864	14948	200	200	0.638
11	**12202**	-124461	16981	14948	-126650	9448	18639	-126809	1381	18639	-124848	9040	14948	200	200	0.637
12	**12401**	-122309	-984	12450	-124836	-7339	14863	-124500	-15286	14863	-122170	-8765	12450	200	200	0.510
13	**12301**	-122218	7020	12450	-124935	836	14863	-124839	-7119	14863	-122312	-765	12450	200	200	0.510
14	**12201**	-121889	15017	12450	-124785	9010	14863	-124931	1053	14863	-122215	7237	12450	200	200	0.510
15	**22101**	-121319	23001	12450	-124388	17176	14863	-124776	9225	14863	-121879	15231	12450	200	200	0.509

Figure 14: Parameters for form finding procedure

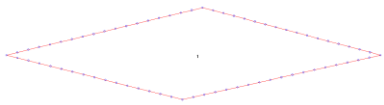
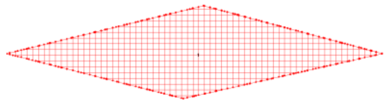
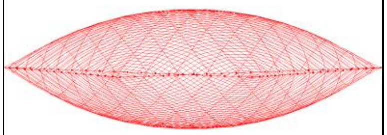
Boundary generation		Net generation		Equilibrium under inner pressure
	→		→	

Figure 15: Step-by-step process of automatic volume form finding based on the given parameters

In a second step, the cutting patterns for the top- and bottom layers were also determined automatically.

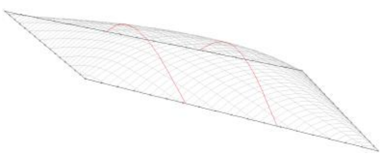
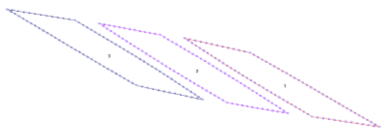
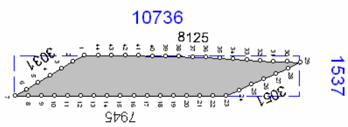
Cutting lines as geodesic lines		Cutting patterns with smallest distortions		Patterns: System line + seam-allowance as dxf file for cutting machines
	→		→	

Figure 16: Step-by-step process of automatic cutting pattern generation based on the given parameters

The second example also shows an ETFE building envelope. This is the Khan Shatyr in Astana (Kazakhstan) and is the largest tent in the world, 150 m high. The structure consists of a cable net with 836 triple-layer ETFE-cushions.

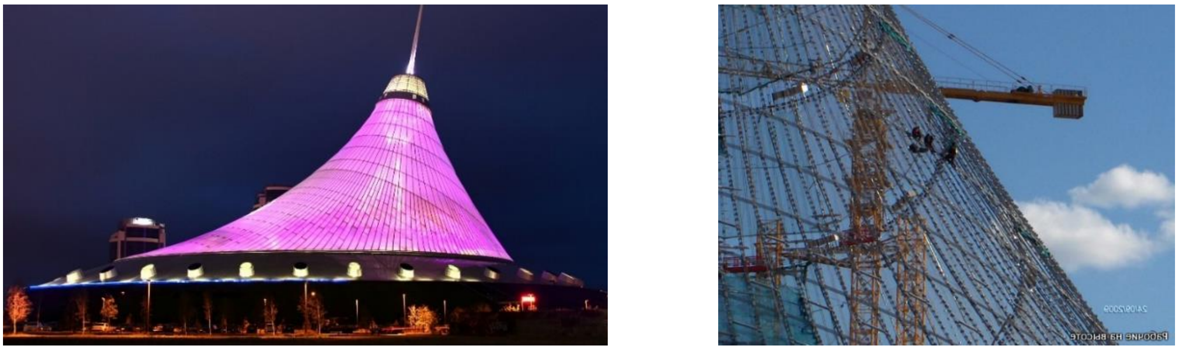


Figure 17: Khan Shatyr in Astana and cable net

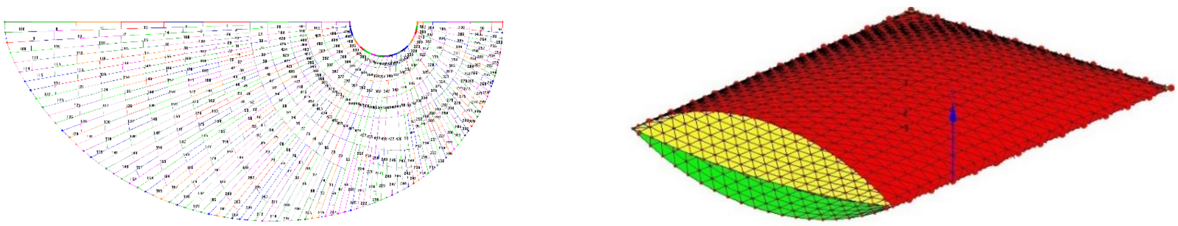


Figure 18: General plan of the cushion fields (left), 3-layer double chamber cushion (right)

In this project, too, the parameters for form finding and cutting pattern generation were read into an automated volume form finding process. The optimization process is shown in the next figure.

The process starts with desired sags (s_{d1}, s_{d2} and s_{d3}) for the 3 foil layers:

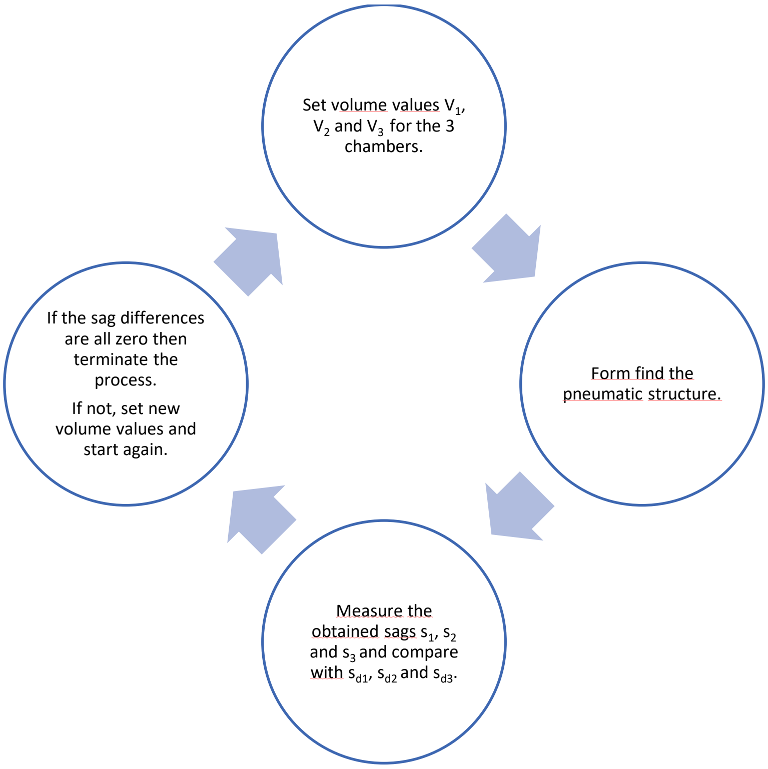


Figure 19: Form finding optimization process

The result was again width-optimized cutting patterns for the 3 layers of the 2-chamber pneumatic cushions. The cutting patterns were generated by the algorithm in such a way that the seam lines of the upper, lower and middle foil were shifted at the edges with a corresponding offset. This has prevented problems due to thickening when clamping the foil in the frame. The cushion skin was realized by vector foiltec together with bfl Tritthardt + Richter. The automated software came from technet GmbH.

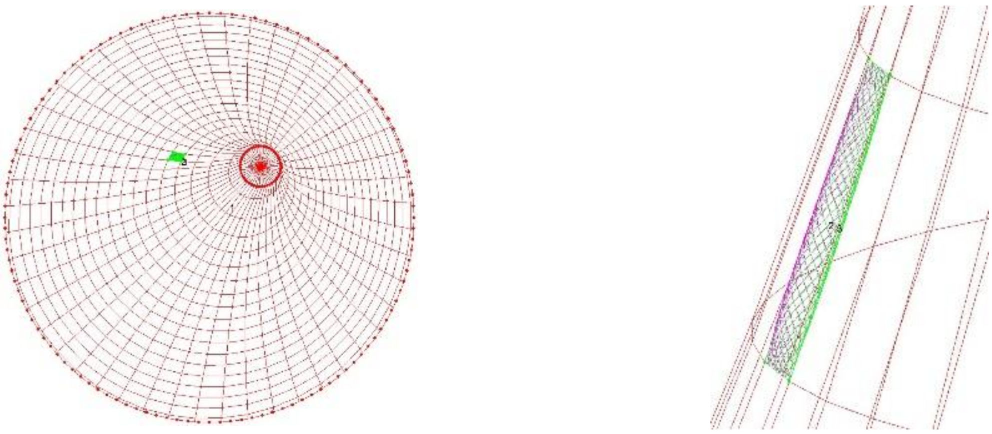


Figure 20: Top view (left) and cable net with 2 chambered cushion (right)

Especially the automatic optimization of the pattern layout is more and more important as for big ETFE-projects mass production of patterns should be managed in a short time and with highest accuracy.

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