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Bending-active frame: analysis and estimation of structural parameters

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Abstract

Bending-active frame, considered in the research, consists of high-strength low-modulus beams, hinged struts and flexible cables. The beams, forming the top chord of the frame, are initially straight. Cable tensioning results in substantial deformation of the beams, and the frame turns into an arch or dome-shaped structure. Specialized software packages of nonlinear structural design require that parameters of the frame would be known in advance, before the analysis. The present work is aimed at estimation of initial span of the frame and the diameter of the top-chord beam. The parameters are checked for compliance with system requirements. The differential equation of a curved beam (the Euler–Bernoulli law for large deformations) is used. The results are verified by comparison with data, provided by the computer software of nonlinear analysis EASY. The present work contributes to the process of design of bending-active structures. It facilitates preliminary calculations required for approval or rejection of a specific design solution. The results of the work may also be used for structural optimization of the bending-active frame in order to obtain its geometrical dimensions and cable tensioning, which correspond to the minimum consumption of material. The work allows to expand the field of application of non-metallic constructions, which require smaller environmental footprint.

Keywords: pre-stressed, flexible beam, fiber reinforced polymer, bending-active

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1. Introduction

The frame, considered in the research, belongs to bending-active structures, which "derive their geometry from the elastic deformation of initially straight elements" (Knippers, Cremers, Gabler and Lienhard, 2011). Bending-active structures with flexible pre-stressed membrane are state-of-the-art constructions having significant potential for possible applications (Lienhard, 2012). The frame, considered in the research, is capable to sustain assembly loads without the membrane attached and the loads caused by the membrane during its hanging, tensioning and operation. It simplifies the construction process in comparison to structures, "which only have sufficient stability through the interaction of stiff and flexible elements" (Seidel, 2009).

The top chord of the frame is made of high-strength low-modulus beams 3, supported by hinged struts and steel cables underneath (figure 1, a) (Chesnokov, Mikhaylov and Dolmatov, 2017). The beams are initially straight. They are situated in a common plane and arranged in parallel or radial directions. Tensioning of cable 1 transforms the frame from its original, so-called "flat", configuration into the operational or dome-shaped one. The joints of the top chord should be implemented according to the specialized design guides, for example (FIBERLINE, 2019). Steel sleeves are to be provided in connections for protection of fiberglass beams.

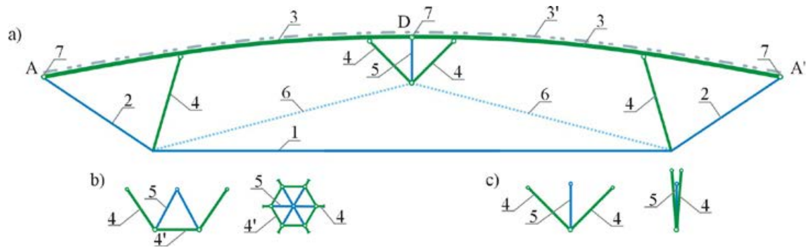

Figure 1: The frame, considered in the research. a – general view; b, c – central lattice structures (different embodiments). 1 – tensioning cable, 2 – diagonal cable, 3, 3' – top chord beams (different embodiments), 4, 4'– hinged struts, 5 – central tie, 6 – additional tie, 7 – hinge at the end of the beam

The outer ends of the beams are pivotally connected to the supports of the frame at points A and A'. Due to high slenderness it is proposed to apply half-span (short) beams 3 linked together by hinges 7 (point D, figure 1, a), rather than full-span (long) beam 3', connecting the opposite supports. It allows to achieve compact size of the frame, prepared for transportation from one site to the other.

In order to rigidly unite the beams, lattice structures of two types are considered (figure 1, b, c). The structures consist of hinged struts 4, 4' and flexible ties 5.

The first embodiment of the lattice structure (figure 1, b) is derived from (Chesnokov, Mikhaylov and Dolmatov, 2015). It improves stability of the top chord in, so-called, out-of-plane direction. On the other hand, flexible polymer membrane, attached to the top chord, performs a similar function preventing compressed members from buckling in the horizontal direction (Mele, Laet, Veenendaal, Mollaert and Block, 2013). The second embodiment or triangular-like lattice structure consists of fewer members. It can be easily adjusted in-situ. The structure of the second type, even if applied in the spatial frame, can be compactly folded (figure 1, c, right), while the ring-shaped bottom chord of the first embodiment (figure 1, b, right) complicates packaging, increasing transportation expenditures.

Non-uniform load may result in substantial deformations of the frame, but installation of additional ties 6 ensures its serviceability reducing the deflections to an appropriate level (Chesnokov, Mikhaylov and Dolmatov, 2017). Unlike direct connection of additional ties 6 to the beams, the lattice structures allow to adjust stresses and moments in the top chord.

The frame, considered in the research, is proposed to be delivered to the construction site in form of particular members: top chord beams 3 with struts 4, the central lattice structure in folded configuration and the cables with anchors. Having assembled the frame, pre-stressing is implemented by means of tensioning devices, such as turnbuckles or threaded rods. When the desired frame shape is reached, the devices are replaced with connecting links (Seidel, 2009).

The present work is aimed at estimation of initial span of the frame and the diameter of the top-chord beam. It allows to facilitate structural analysis of the frame using specialized software packages of nonlinear design and contributes to the process of optimization in order to achieve minimum consumption of material. The parameters of the frame are checked for compliance with system requirements, such as the strength properties and the conditions of compressive buckling of the top-chord. The differential equation of a curved beam is used.

2. Parameters of the frame

It is assumed in the research, that the beams of the frame are of a tubular cross section, and the pre-stress of the frame is performed by means of cable 1 tensioning. The plain model of the frame is taken into account. Parameters, considered in the research, may be broken down into the following groups:

– geometrical dimensions (given parameters are marked with red color in figures 2 and 3): initial frame span S_0 and the span S_1 of the frame in the operational state, horizontal dimensions a_0, b_0, c_0 and angles between the frame members α_0, β_0 and γ_0;

– tensioning of cable 1 ΔL_p ;

– material properties: the modulus of elasticity and the strength of the beam and the cables - E_b , R_b and E_{cab} , R_{cab} , respectively;

– properties of the frame members: cross section area of the cables A_{cab} , diameter of the beam D_b and thickness-to-diameter ratio k_t .

Material properties are assumed to be given in advance. According to (PFEIFER, 2017) the properties of steel cables are the following: $E_{cab} = 130$ GPa and $R_{cab} = 700$ MPa. The beams should be made of materials with the ratio $R_b / E_b \geq 2.5$ (Lienhard, Alpermann, Gengnagel and Knippers, 2013). Pultruded glass-fiber reinforced polymer (FIBERLINE, 2019) is an appropriate material for this purpose due to its superior properties: $E_b = 24$ GPa and $R_b = 185$ MPa. These values are, however, for short-term use only (Kotelnikova-Weiler, Douthe, Hernandez, Baverel, Gengnagel and Caron, 2013).

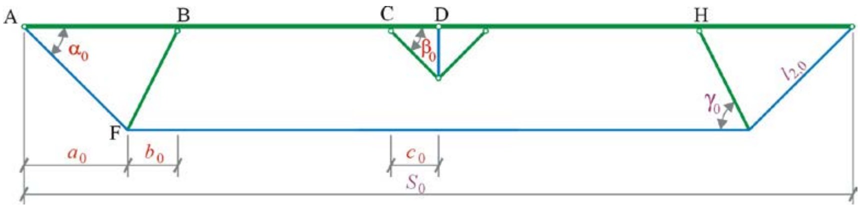

Figure 2. Geometrical parameters of the frame

The following condition, regarding to the linear dimensions of the frame, may be written:

$$a_0 + b_0 + c_0 \leq S_0 / 2 \quad (1)$$

Angle dimensions α_0 and β_0 are assumed to be in the range $[20...70]^0$. Low bound for the angle α_0 is also determined by the following condition:

$$\alpha_0 - \Delta\alpha > \alpha_{\lim} \quad (2)$$

where $\Delta\alpha$ is the angle alteration by the frame transformation into the operational state (figure 3), $\alpha_{\lim}$ is a given limiting value for the angle α .

3. Determination of the initial frame span S_0

The span of the frame in the operational state S_1 is considered to be given. Thus, the span S_0 is to be determined in order to obtain the initial lengths of the frame members.

It is approximately assumed, that the triangle ABF rotates like a rigid body, while the internal part of the frame, situated between points B and H, takes on parabola-like shape with the span S_p and the rise f_p (figure 3).

The length of the parabola is determined by the following simplified expression:

$$L_p = S_p + \frac{8}{3} \cdot \frac{f_p^{\,2}}{S_p} \quad (3)$$

It is approximately equal to the unstressed length of the beam parts, situated between points B-D and H-D:

$$L_p \approx S_0 - 2 \cdot (a_0 + b_0) \quad (4)$$

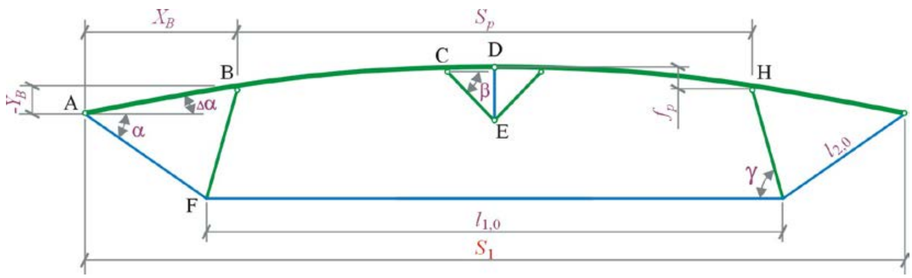

Figure 3. The frame configuration in the operational state

Line AB passes along the tangent to the parabola at the end point:

$$\tan(\Delta\alpha) = 4 \cdot \frac{f_p}{S_p} \quad (5)$$

Substituting the ratio f_p / S_p from (5) into (3) gives:

$$\cos(\Delta\alpha) = \frac{1}{\sqrt{6 \cdot L_p / S_p - 5}} \quad (6)$$

On the other hand:

$$\cos(\Delta\alpha) = \frac{S_1 - S_p}{2 \cdot (a_0 + b_0)} \tag{7}$$

Substituting (4) and (7) into (6), the dependence for the initial span of the frame S_0 is derived:

$$S_0 = \left[2 \cdot \left(\frac{a_0 + b_0}{S_1 - S_p} \right)^2 + 2.5 \right] \cdot \frac{S_p}{3} + 2 \cdot (a_0 + b_0) \tag{8}$$

The expression for the span S_1 is the following:

$$S_1 = l_{1,0} + 2 \cdot l_{2,0} \cdot cos(\alpha_0 - \Delta\alpha) \tag{9}$$

where $l_{1,0}$ and $l_{2,0}$ are the initial lengths of the cables, forming the bottom chord (figure 3): $l_{1,0} = S_0 - 2 \cdot a_0 - \Delta L_p$, $l_{2,0} = a_0 / \cos(\alpha_0)$.

Substituting (8) into (9), considering (7), the following equation in one unknown is derived:

$$\Theta(\Delta\alpha) = S_1 + \Delta L_p - 2 \cdot b_0 \tag{10}$$

where

$$\Theta(\Delta\alpha) = 2 \cdot a_0 \cdot (\cos(\Delta\alpha) + \tan(\alpha_0) \cdot \sqrt{1 - \cos(\Delta\alpha)^2}) + \left(\frac{1}{\cos(\Delta\alpha)^2} + 5 \right) \cdot \frac{S_1 - 2 \cdot (a_0 + b_0) \cdot \cos(\Delta\alpha)}{6} \tag{11}$$

Having obtained the angle $\Delta\alpha$ from (10) by a diagram or numerically, the initial span of the frame S_0 is calculated from (8), considering (7).

4. Estimation of the diameter of the beam

4.1. Structural behavior of the beam

The shape of the beam may be approximately represented as follows (figures 3, 4):

– segment A-B ($0 \le x < X_B$, $X_B = (a_0 + b_0) \cdot \cos(\Delta\alpha)$):

$$Y(x) = -x \cdot \tan(\Delta\alpha) \tag{12}$$

– segment B-D ($X_B \le x \le X_D$, $X_D = 0.5 \cdot S_1$ and $Y_B = -(a_0 + b_0) \cdot \sin(\Delta\alpha)$):

$$Y(x) = \frac{4 \cdot f_p}{S_p^{\ 2}} \cdot (x - X_B) \cdot (x - X_B - S_p) + Y_B \tag{13}$$

The differential equation of a beam, subjected to bending, is the following (Fertis, 2006):

$$\frac{Y''(x)}{(1 + Y'(x)^2)^{1.5}} = -\frac{M(x)}{E_b \cdot I_b} \tag{14}$$

where $M(x)$ is a bending moment at point x of the beam; I_b is the second moment of area of the beam.

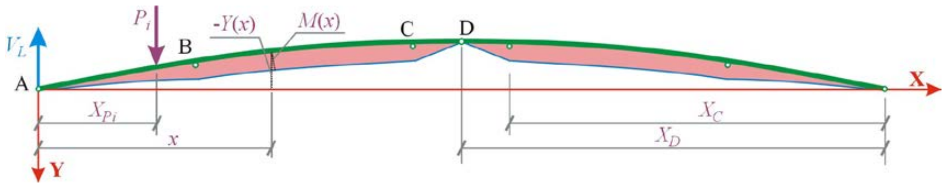

Figure 4. Schematic diagram of bending moments in the beam

Diagram of bending moments in the beam of the frame is shown in the figure 4. In case of symmetric load distribution, the moments are also symmetric relatively to the vertical axis, passing through the point D. Expressions for the moments $M(x)$ are the following:

– segment A-B ($0 \le x < X_B$):

$$M_{A-B}(x) = m_{ab}(x) \tag{15}$$

where $m_{ab}(\xi) = N_2 \cdot (Y(\xi) \cdot \cos(\alpha) - \xi \cdot \sin(\alpha)) + M_{LD}(\xi)$; N_2 is the force in the cable 2 (figure 1); $M_{LD}(\xi)$ is the bending moment, brought about by external loads P_i situated on the left relatively to the corresponding section ξ of the beam:

$$M_{LD}(\xi) = V_L \cdot \xi - \sum_i P_i \cdot (\xi - X_{Pi}) \tag{16}$$

where V_L is the vertical support reaction, brought about by the external loads; X_{Pi} is the load coordinate, $X_{Pi} < \xi$; $\alpha = \alpha_0 - \Delta\alpha$ is the angle between the cable and the horizontal axis;

– segment B-C ($X_B \le x < X_C$, $X_C = 0.5 \cdot S_1 - c_0$):

$$M_{B-C}(x) = m_{bc}(x) \tag{17}$$

where $m_{bc}(\xi) = m_{ab}(\xi) + N_2 \cdot \sin(\alpha) \cdot \left[(Y(\xi) - Y(X_B))/\tan(\gamma) + \xi - X_B\right]$; $\gamma = \gamma_0 + \Delta\alpha$;

– segment C-D ($X_C \leq x \leq X_D$):

$$M_{C-D}(x) = m_{bc}(x) + m_{bc}(X_D) \cdot \frac{X_C - x}{X_D - X_C} \tag{18}$$

Maximum stress in the beam σ_b must be less than the strength of the material:

$$\sigma_b < R_b \tag{19}$$

The stress is expressed according to (FIBERLINE, 2019) as follows:

$$\sigma_b = \frac{N_b}{A_b} + \frac{1}{1 - \frac{N_b}{N_{cr}}} \cdot \frac{|M_{b,\max}|}{W_b} \tag{20}$$

under the condition:

$$N_b < N_{cr} = \frac{F_d}{1 + F_d / N_{el}} \tag{21}$$

where $F_d = A_b \cdot R_b$ and N_{el} is the Euler load $N_{el} = \pi^2 \cdot E_b \cdot I_b / (K \cdot L_u)^2$, where K is the effective length factor and L_u is the length of a half of an arch, formed by the curved beam (EN 1993-2:2006).

The maximum moment $M_{b,\max}$ is obtained from (15), (17), (18), while the axial force in the beam N_b, the second moment of area I_b, the cross section area A_b and the elastic section modulus W_b are expressed as follows:

$$N_b = N_2 \cdot k_b \tag{22}$$

where $k_b = \sin(\alpha_0 + \gamma_0) / \sin(\gamma_0 + \Delta\alpha)$ and

$$I_b = \frac{\pi \cdot D_b{}^4}{8} \cdot k_t \cdot (1 - k_t) \cdot [2 \cdot k_t \cdot (k_t - 1) + 1] \tag{23}$$

$$A_b = I_b \cdot \frac{8}{D_b{}^2 \cdot [2 \cdot k_t \cdot (k_t - 1) + 1]} \tag{24}$$

$$W_b = \frac{2 \cdot I_b}{D_b} \tag{25}$$

4.2. Estimation of the upper bound for the diameter of the beam

Lets consider the stage of the frame pre-stress. In the assumption, that own weight of the frame is negligible, external loads P_i are omitted and $M_{LD}(x)=0$. Thus, bending moments in the beam may be expressed as follows:

$$M(x)=M_1(x)\cdot N_{2,pr} \tag{26}$$

where index "*pr*" refers to the stage of the frame pre-stress; $M_1(x)$ is the moment in *x*-section of the beam (15), (17), (18), considering the force in the cable $N_2=1$.

The differential equation (14) may be written for the point B as follows:

$$\frac{8\cdot f_p\cdot S_p}{\left[S_p{}^2+16\cdot f_p{}^2\right]^{3/2}}=\frac{-M_1(X_B)}{E_b}\cdot\mu \tag{27}$$

where μ is the ratio:

$$\mu=N_{2,pr}/I_b \tag{28}$$

Having obtained μ-value from (27), the upper bound for the diameter of the beam $D_b<D_{\lim,up}$ is expressed from the conditions (19) and (21) as follows:

$$D_{\lim,up}=\min\left\{\frac{-\lambda_1\cdot\mu+\sqrt{(\lambda_1\cdot\mu)^2+8\cdot\lambda_2\cdot\mu\cdot R_b}}{2\cdot\lambda_2\cdot\mu},\ \sqrt{\frac{8\cdot R_b}{2\cdot k_t\cdot(k_t-1)+1}\cdot\left[\frac{1}{\mu\cdot k_b}-\frac{1}{E_b}\cdot\left(\frac{K\cdot L_u}{\pi}\right)^2\right]}\right\} \tag{29}$$

where

$$\lambda_1=\frac{|M_{1,\max}|}{1-\left(\frac{K\cdot L_u}{\pi}\right)^2\cdot\frac{\mu\cdot k_b}{E_b}}\quad\text{and}\quad\lambda_2=\frac{k_b}{4}\cdot[2\cdot k_t\cdot(k_t-1)+1] \tag{30}$$

where $M_{1,\max}$ is the maximum moment in the beam, considering $N_2=1$ and $M_{LD}(x)=0$.

4.3. Estimation of the lower bound for the diameter of the beam

The lower bound for the diameter of the beam $D_b\geq D_{\lim,low}$ is proposed to be found in the step-by-step manner. Having started from the minimum permissible value $D_{b,\min}$, the diameter of the beam D_b is incremented until the conditions (19) and (21) are fulfilled and the lower bound is reached $D_{\lim,low}=D_b$. If, however, $D_{\lim,low}>D_{\lim,up}$, the set of structural parameters, such as geometrical dimensions, cable tensioning and material properties, is not permissible for the given value of external load.

The diagonal cable force $N_{2,Ld}$, needed to calculate the force N_b and the moment $M_{b,\max}$ in the beam, is obtained from the following expression:

$$N_{2,Ld}(D_b) = N_{2,pr}(D_b) + \Delta N_2 \tag{31}$$

where $N_{2,pr}(D_b)$ is expressed from (28), considering (23); ΔN_2 is the force alteration, brought about by the external load:

$$\Delta N_2 = \frac{-\int_0^{X_D} M_1(x)\cdot M_{LD}(x)\cdot\sqrt{1+\left(\frac{d}{dx}Y(x)\right)^2}\,dx}{\int_0^{X_D}[M_1(x)]^2\cdot\sqrt{1+\left(\frac{d}{dx}Y(x)\right)^2}\,dx} \tag{32}$$

where $Y(x)$ is the shape function of the beam, given by (12) and (13).

The expressions (31) and (32) are obtained by the flexibility method, which is valid for small shape alterations of the frame in case of symmetrical external loading.

5. Case study

The frame with the following parameters is considered as an example: the span $S_1 = 12.0$ m, horizontal dimensions $a_0 = 1.5$, $b_0 = 0.0$, $c_0 = 1.0$ m, angles between the frame members $\alpha_0 = 45^0$, $\beta_0 = 45^0$ and thickness-to-diameter ratio of the tubular beam $k_t = 1/20$. Tensioning of the cable 1 is assumed $\Delta L_p = 1.0$ m. Uniformly distributed external load $q_{LD} = 0.5$ kN/m is taken into account for operational stage of the frame.

From the equation (10), considering (11), the angle alteration is obtained the following $\Delta\alpha = 16.7^0$, the span of the parabola is calculated from (7) $S_p = 9.127$ m and the initial span of the frame is obtained from (8) $S_0 = 12.263$ m. Equation (27) allows to determine μ-value, $\mu = 1.307\cdot 10^6$ kN/m^4, while the upper bound for the diameter of the beam is obtained from (29) $D_{\lim,up} = 0.153$ m, considering the effective length factor $K = 1.04$ (EN 1993-2:2006) (two-hinged arch with rise-to-span ratio 0.1) and the length of a half of the arch $L_u = S_0/2 = 6.132$ m.

Having calculated the force alteration $\Delta N_2 = 5.33$ kN, considering (17), (20), (22)-(25) and (31), the appropriate beam diameter $D_b = 0.125\,\text{m}$ is found from the conditions (19) and (21): $\sigma_b / R_b = 0.99$ and $N_b / N_{cr} = 0.35$.

In order to verify results, obtained by the proposed expressions, numerical simulation of the bending-active frame is performed by means of the software package of structural analysis EASY. After the stage of the pre-stress, the span of the frame is $S_{1,EAS} = 12.025\,\text{m}$ (discrepancy is $\varepsilon = 0.2\,\%$), the angle alteration $\Delta\alpha_{EAS} = 17.3^0$ ($\varepsilon = 3.6\,\%$) and the distance between points B and H (figure 3) is $S_{p,EAS} = 9.159\,\text{m}$ ($\varepsilon = 0.4\,\%$), where index "*EAS*" refers to results, obtained by the EASY software and ε is the relative discrepancy between the proposed results and the results by EASY. The force alteration, caused by the external load, is $\Delta N_{2,EAS} = 5.3$ kN ($\varepsilon = 0.6\,\%$). The conditions (19) and (21) are fulfilled as well: $\sigma_{b,EAS} / R_b = 0.93$ ($\varepsilon = 6.5\,\%$) and $N_{b,EAS} / N_{cr} = 0.36$ ($\varepsilon = 2.9\,\%$).

6. Conclusion

Light-weight bending-active frame is considered in the research. It requires less expenditures for transportation and installation, than ordinary building constructions. The frame is intended for temporary sheltering of areas of social occasions, entertaining events, points of retail, bus stations, etc.

In accordance to (Lienhard, Alpermann, Gengnagel and Knippers, 2013) the proposed work belongs to, so-called, "geometry based approach", which uses analytical techniques for structural simulation of bending-active systems. On the other hand, it provides essential data for rapidly developing "integral" approach, which is based on specialized software packages of nonlinear structural design.

The present work contributes to the development of bending-active structures. It facilitates the conceptual design stage, allowing to consider a lot of competing design options and to select the best one. The results of the work may also be used for structural optimization of the bending-active frame. The work allows to expand the field of application of non-metallic constructions, which require smaller environmental footprint.

The future work should take into account arbitrary external load distribution, including loads in the horizontal direction. Time-dependent material behavior and temperature variations during the operational period are to be considered.

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