SOFTENING THE HABITATS
SUSTAINABLE INNOVATION IN MINIMAL MASS STRUCTURES AND LIGHTWEIGHT ARCHITECTURES

Proceedings of the TensiNet Symposium 2019
Softening the habitats | 3-5 June 2019, Politecnico di Milano, Milan, Italy
Alessandra Zanelli, Carol Monticelli, Marijke Mollaert, Bernd Stimpfle (Eds.)

Reliability-based analysis of a cable-net structure designed using partial factors

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Abstract

The design of tensioned membrane structures is not yet addressed by standards like the Eurocode. Currently, in cooperation with the TensiNet Association, Working Group 5 of CEN TC250, is working on a general standardised design approach for membrane structures, consistent with the partial factor framework used in the Eurocode (EN1990). To achieve such design approach, research towards a method to verify the reliability of a membrane structure is needed. The structural behaviour of a tensioned membrane can be represented by a tensioned cable-net. Herein a cable-net structure is firstly investigated because cables are addressed by the Eurocode (EN1993-1-11). Further research will investigate the established method for membrane structures. This paper explains and evaluates the influence of the partial factor for pretension. If the cables are dimensioned according to a partial factor 1.0, the obtained reliability indexes are lower than the considered target reliability index. If the cables are dimensioned according to a partial factor 1.35 the obtained reliability indexes are higher than the considered target reliability index.

Keywords: reliability index, cable-net, Latin Hypercube Sampling.

DOI: 10.30448/ts2019.3245.18

Peer-review under responsibility of the TensiNet Association

1. Introduction

Since the 1950-s, the tensile surface structure industry has emerged and evolved, resulting in the wide variety of membrane structures and membrane materials available today. Although the membrane structure concept is generally approved, the design of membrane structures is not yet addressed by current standards like the Eurocode. Working Group 5 of CEN TC250 is working on a general standardised design approach for membrane structures. To obtain a design procedure that can be used by architects, engineers and contractors, such as the partial factor framework used in the Eurocode (EN1990), some partial factors need to be proposed and evaluated for membrane structures. So, research towards reliability analysis in combination with membrane structure analysis is needed to obtain such framework.

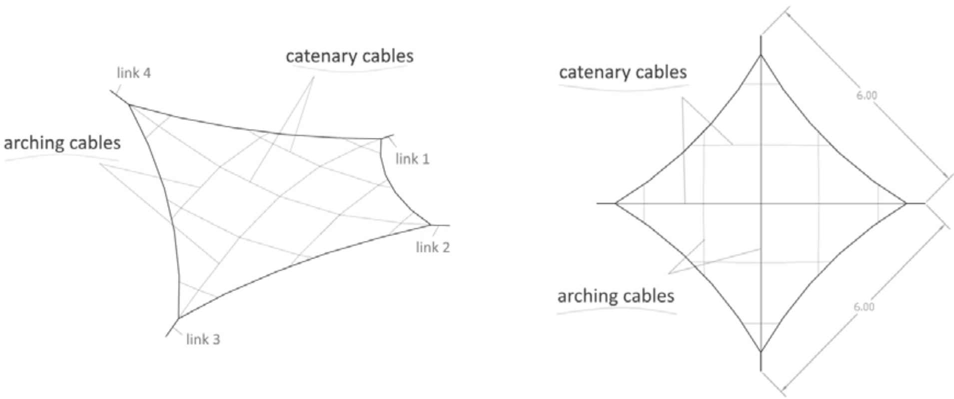
Gosling et al. (Gosling, Bridgens, & Zhang, 2013) and Thomas et al. (Thomas, Schoefs, Caprani, & Rocher, 2015) already studied the combination of reliability analysis with membrane structural analysis. Gosling et al. proposed and evaluated a method to perform a reliability analysis for membrane structures. The method consists of a sensitivity analysis and a reliability analysis based on the most important uncertain parameters. The found reliability indexes are lower than the target reliability indexes specified in the Eurocode. They concluded that the method is applicable for membrane structures if sufficient information is available about the stochastic models of the parameters. Thomas et al. investigated the reliability analysis of an inflated beam. The authors concluded that the obtained reliability indexes are in line with the target reliability indexes in the Eurocode.

The study presented in this paper also investigates a method to estimate the reliability of a membrane structure. The overall focus of the paper is on the used method. The structural behaviour of a tensioned membrane can be represented by a tensioned cable-net. Steel cables are addressed by the Eurocode (EN1993-1-11) and therefore a cable-net structure is firstly investigated and discussed in this paper. For the reliability analysis, the first order reliability method (FORM) (Sørensen, 2004) is used in combination with Latin Hypercube Sampling (LHS) (Olsson, Sandberg, & Dahlblom, 2003).

Further research will investigate the established method for membrane structures (ongoing research Round Robin exercise IV (De Smedt et al., 2018)). This paper explains and evaluates the influence of the partial factor for pretension. Therefore, the cable-net is designed with a partial factor for pretension of 1.0 and 1.35. Afterwards the reliability index β is calculated for both structures under snow load and under wind uplift load. The index is calculated for the catenary as well as for the arching cables.

2. Description of the cable-net

The studied cable-net structure is a hyperbolic paraboloid with five catenary cables, five arching cables, boundary cables and a turn-buckle in each of the four corner-points (Figure 1).



The in-plane distance from high point to low point is 6 meters.

Figure 1: Perspective view (left) and top view (right) of the cable-net.

Steel cables are used for the cable-net. The cross-section is calculated as follows (CEN, 2006):

$$A_{cc/ac} = \frac{N_{max,cc/ac}}{f_d} \quad (1)$$

$$f_d = \frac{f_u}{1.5\gamma_m} \quad (2)$$

where $A_{cc/ac}$ = area of the catenary and the arching cables; $N_{max,cc/ac}$ = maximum axial force in the catenary and the arching cables; f_d = design strength of the material; f_u = ultimate tensile strength = 1770 N/mm² (CEN, 2006); γ_m = material safety factor for steel (= 1.0).

Note that the cross-section is optimally dimensioned and considered as one circular section (so no bundle of steel wires like used for real steel cables). The structure is subjected to snow load and wind uplift load. For the dimensioning two cases are considered:

Case 1: 1 x pretension + 1.5 x snow load,

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Case 2: 1.35 x pretension + 1.5 x snow load,

1.35 x pretension + 1.5 x wind uplift load.

The partial factors are according to the Eurocode and correspond to a target reliability index of 3.8 for a 50 years reference period and a reliability class two (Table 1) (CEN, 2001).

Table 1: Target reliability indexes according to Eurocode, for a 1 year and a 50 years reference period.

Reliability class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5.2	4.3
RC2	4.7	3.8
RC1	4.2	3.3

Table 2 shows the values for the different loads. The values for snow load and wind uplift load are in line with the Eurocode.

Table 2: Values for pretension, snow load and wind uplift load.

	Value loads	Unit
Pretension	7.5	kN
Snow load	-0.6	kN/m²
Wind uplift load	1	kN/m²

The focus of this study is to evaluate a reliability analysis method and not to reach the safety requirements specified in the Eurocode. Therefore, the pretension of the cable-net is 7.5 kN for both directions so that a minimum tension of ± 0.75 kN remains present in both cable groups under loading.

The cross-section of both cable-groups for case 1 and 2 are given in Table 3.

Table 3: Cross-section of the catenary cables (CC) and the arching cables (AC) for case 1 and 2.

	CC	AC	Unit
Case 1	10.51	13.14	mm²
Case 2	12.85	15.64	mm²

As expected, the cross-section of both catenary and arching cables increases when the partial factor for pretension is increased from 1.0 to 1.35.

3. Reliability analysis

There are four methods to measure the reliability of a structure or structural element, called level I to IV methods. Level I is the partial factor approach used in the Eurocode (CEN, 2001), level II uses the mean values and the standard deviation of the uncertain parameters, level III uses the joint distribution functions and level IV incorporates the costs of failure (economic aspect) (Sørensen, 2004). The reliability analysis in this study is performed by level II methods together with first order reliability method (FORM). FORM means that the limit state function is linearized (Sørensen, 2004). In this study, the considered limit state function is ULS, considering the resistance R of the material and the effect E (CEN, 2001):

$$z = R - E \quad (3)$$

Figure 2 visualises the failure surface $z = 0$, the reliability index β and the design point P in a standard space u. The standard space is introduced by Hasofer and Lind (Hasofer & Lind, 1974). The variables R and E are transformed to respectively U_R and U_E.

$$U_R = \frac{R-\mu_R}{\sigma_R} \quad (4)$$

$$U_E = \frac{E-\mu_E}{\sigma_E} \quad (5)$$

where $\mu_{R/E}$ is the mean value of R and E and $\sigma_{R/E}$ is the standard deviation. The straight, inclined line in Figure 2 is called the failure surface. The surface represents where the limit state function z is zero. The structure is in the safe state when z is larger than zero (right of the failure surface) and in the failure state when z is smaller than zero (left of the failure surface). To measure the reliability of a structure, the reliability index can be calculated:

$$\beta = \frac{\mu_z}{\sigma_z} \quad (6)$$

where μ_z is the mean value of z and σ_z is the standard deviation.

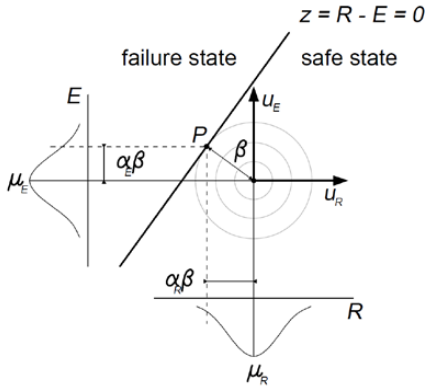

Figure 2: Visualisation limit state function *z*, design point *P* and reliability index β (CEN, 2001).

Design point P is the indication on the failure surface wherefore failure of the structural element or structure is the most probable. The distance from this design point to the origin is the representation of the reliability index, which is the smallest distance from the origin to the failure surface (CEN, 2001).

2.1. Latin Hypercube Sampling

A reliability analysis is performed by taking into account the stochastic models of the uncertain parameters (Sørensen, 2004). This means that a range of values of the parameters based on their mean value, coefficient of variance (COV), etc., has to be evaluated. In order to obtain the reliability index, different simulation sets containing variations of the uncertain parameters are put together. The sets are established by means of Latin Hypercube Sampling (LHS) (Mckay M.D. et al. 2000). The method allows to estimate the reliability of a structure with a limited amount of simulations. In Figure 3, a schematic representation of the LHS concept on a cumulative density function (CDF) is shown. The distribution of the parameter is divided in a certain number of equal parts (grey lines) and from each part one value is randomly taken (black crosses). Like this, the whole distribution of the parameter is taken into account.

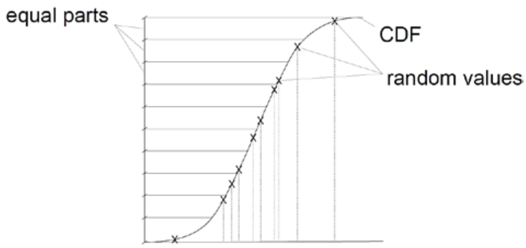

Figure 3: Schematic representation of the LHS concept with a CDF.

2.2. Parameters

The considered parameters and their stochastic models (the type of distribution, the nominal value, the mean value, the standard deviation and the COV) are given in Table 4.

Table 4: The stochastic models of the considered parameters.

	RA 1/RA 2	Distribution	Nominal values	μ	σ	COV	unit
l_{1-4}	1, 2	Normal	0,5	0,5	0,002	0.004	m
EA_{cc}*	1, 2	Normal	1734	1734	34,68	0.020	kN
EA_{ac}*	1, 2	Normal	2168	2168	43,36	0.020	kN
f_u	1, 2	Normal	1770	1830	30	0.016	MPa
Q_{snow}	1	Gumbel	-0,6	-0,66	-0,20	0.300	kN/m²
Q_{wind}	2	Gumbel	1	0,7	0,25	0.350	kN/m²

In total, eight parameters are considered: the length of each turn-buckle l_{1-4}, the axial stiffness of both cable groups $EA_{cc/ac}$ (catenary and arching cables), the ultimate strength of the steel cables and the load $Q_{snow/wind}$. The lengths of the turn-buckles are considered as an installation uncertainty. During construction, the pretension of the cable-net is set by means of the turn-buckles, that are adjusted in length on the construction site. Variation on the lengths affects the geometry and the pretension of the cable-net, which means that there is an indirect uncertainty on the geometry and the pretension. The axial stiffness of the catenary and the arching cables is the modulus of elasticity of the steel cables multiplied with the area of the cross section of each cable group:

$$Stiffness = EA_{cc/ac} \tag{7}$$

where E = modulus of elasticity = 165 x 10^3 MPa (Krishna P. 1978). The stochastic models of the lengths of the turn-buckles and of the stiffness of the cables are based on expert advice. According to expert advice, a variation of pretension of 0.75 kN/m is to be expected after installation of the cable-net. The COV of the length of the turn-buckles is chosen so that the expected variation of pretension is met.

Due to the lack of available data of these stochastic models, the normal distributions are a straightforward choice for this first reliability analysis of a tensioned cable-net structure. Normal distributions can imply negative values. It proved not to have a significant influence on the outcome for this case study. The models of the yield strength, snow load and wind uplift load are taken from a publication of the Joint Committee on Structural Safety (JCSS) (Holicky & Sykora, 2010).

The length of the turn-buckles, the stiffness and load values are used in the simulations. The ultimate strength is used in the limit state function. For this study 100 simulations proved to be sufficient to perform an accurate reliability analysis.

4. Results

The reliability analysis is performed by FORM and LHS in combination with the form finding and structural analysis software Easy (*Easy*, 2017). After the simulations are completed, the reliability is measured with the results of each set according to:

$$z = N_{red,cc/ac} - N_{max,cc/ac} \tag{8}$$

$$N_{red,cc/ac} = f_{red} A_{cc/ac} \tag{9}$$

$$f_{red} = \frac{f_u}{1.5} \tag{10}$$

where $N_{red,cc/ac}$ is the reduced tensile force of respectively the catenary and the arching cables; f_{red} is the reduced tensile strength of the steel cables. The distribution of the resistance and the effect of both cable groups are visualised in Figure 4 and Figure 5.

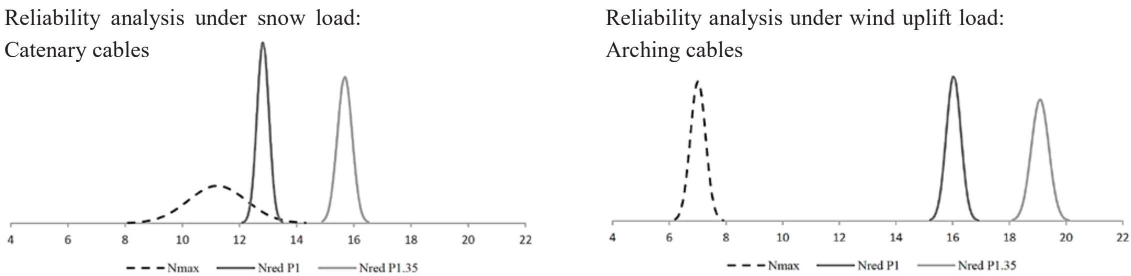

Figure 4: Distributions of the resistance and the effect for the catenary (left) and the arching cables (right) under snow load.

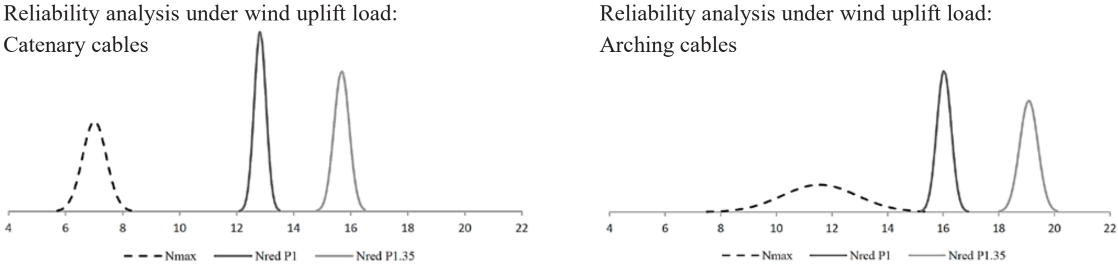

Figure 5: Distributions of the resistance and the effect for the catenary (left) and the arching cables (right) under wind uplift load.

Figure 4 shows the results of the reliability analysis under snow load and Figure 5 under wind uplift load. For the structural dominant direction (CC under snow load and AC under wind uplift load) the 'distance' between the distribution of the resistance and the effect is smaller than for the non-dominant direction. For the dominant cables dimensioned according to a pretension factor 1.0 there is an overlap of the distribution of the resistance and the effect, which means that the cables do not reach the specified safety requirements.

The reliability of the cable-net structure is determined with the lowest reliability index. The obtained reliability indexes obtained for the structural dominant cables under snow load (catenary cables) and wind uplift load (arching cables) are given in Table 5.

Table 5: The obtained reliability indexes for CC and AC dimensioned with a factor 1 and 1.35 for the pretension.

		Snow load CC	Wind uplift load AC
Case 1	1 x pretension	1.52	3.21
Case 2	1.35 x pretension	4.01	5.61

As expected, from Figure 4 and Figure 5, the target reliability index of 3.8 is not reached for the structural dominant cables designed with a pretension factor 1, which is 1.52 under snow load and 3.21 under wind uplift load. For both case 1 and 2 the reliability index is lower for the reliability analysis under snow load than under wind uplift. This is most likely due to the fact that the simulations for the reliability analysis are performed with the mean values of the snow load and wind uplift load (respectively 0.66 kN/m² and 0.7 kN/m², Table 4) which is for the snow load higher than the nominal value (0.6 kN/m²) and for the wind load lower than the nominal value 1 kN/m²) wherefore the cable-net is dimensioned.

For case 1, the reliability index under wind uplift load for the arching cables (3.21) and for case 2, the reliability index under snow load for the catenary cables (4.01) are in line with the specified target reliability index (3.8).

From Table 5 it is noticed, that for this specific steel cable-net structure, the cables dimensioned according to a pretension factor 1.35 meet the safety requirements and are safe according to the considered limit state function (equation (8)).

5. Conclusion

This study evaluates a method to estimate the reliability of a tensioned cable-net structure. The cable-net is designed according to the partial factor method described in the Eurocode. The steel cable-net structure is dimensioned according to two partial factors for pretension 1 and 1.35. The cross-section increases from 10.51 to 12.85 mm² for the catenary cables and 13.14 to 15.64 mm² for the arching cables. The increase of the cross-section results in an increase of the reliability index. The reliability index for the structural dominant direction under snow (CC) increases from 1.52 to 4.01 and under wind uplift load (AC) from 3.21 to 5.61. As according to the Eurocode the partial factor for a permanent load should be 1.35, the reliability index obtained in case 2 is confirmed to be larger than the target reliability index 3.8. The safety requirements are met if the cable-net is designed according to a pretension factor 1.35.

The proposed reliability analysis method proofs to be a valid tool to estimate the reliability of a tensioned cable-net structure. The method is very generic, because it allows inserting the parameters specific to the case study and therefore it can easily be applied to other cable-net structures. A next step is to evaluate the described method for the reliability analysis on a membrane structure and to calibrate the partial factors, which are not yet available in current standards.

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